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# Mu-tau reflection symmetry with a texture-zero

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**ABSTRACT:** The  $\mu\tau$ -reflection symmetry is a simple symmetry capable of predicting all the unknown CP phases of the lepton sector and the atmospheric angle but too simple to predict the absolute neutrino mass scale or the mass ordering. We show that by combining it with a discrete abelian symmetry in a nontrivial way we can additionally enforce a texture-zero and obtain a highly predictive scenario where the lightest neutrino mass is fixed to be in the few meV range for two normal ordering (NO) solutions or in the tens of meV in one inverted ordering (IO) solution. The rate for neutrinoless double beta decay is predicted to be negligible for NO or have effective mass  $m_{\beta\beta} \approx 14\text{--}29$  meV for IO, right in the region to be probed in future experiments.

**KEYWORDS:** CP violation, Neutrino Physics, Beyond Standard Model

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## 1 Introduction

After the discovery of the nonzero value of the reactor angle  $\theta_{13}$  in 2012 [1–3], a few unknowns remain in the neutrino sector if neutrinos are Majorana: the ordering of neutrino masses, the absolute scale of neutrino masses and the values of the three CP phases — one of Dirac type and two of Majorana type. Major experimental efforts on neutrino oscillations are now focused on determining the Dirac CP phase and the mass ordering.

One of the simplest symmetries that can predict all the CP phases and yet allow CP violation is the symmetry known as  $\mu\tau$ -reflection symmetry or  $\text{CP}^{\mu\tau}$  symmetry where the neutrino sector is invariant by exchange of the muon neutrino with the tau antineutrino [4–6]. This symmetry ensures that the Dirac CP phase is maximal ( $\delta = \pm 90^\circ$ ) while the Majorana phases are trivial allowing the possibility of discrete choices of the CP parities. Additionally, the atmospheric angle  $\theta_{23}$  is predicted to be maximal ( $45^\circ$ ) at the same time that  $\theta_{13}$  is permitted to be nonzero. These values for the neutrino parameters are still allowed by current global fits and in fact there are hints that  $\delta \sim -90^\circ$  [7, 8]. The fixed values for the CP phases also lead to characteristic bands for the possible effective mass of neutrinoless double beta decay but still allows leptogenesis to occur if flavor effects are taken into account [9].

It was shown in ref. [9] that the simplest way of implementing  $\text{CP}^{\mu\tau}$  and guarantee diagonal charged lepton masses is to combine  $\text{CP}^{\mu\tau}$  with the combination  $L_\mu - L_\tau$  of family lepton numbers. This combination is trivial in the sense that these two symmetries commute and this feature allows us to avoid the vev alignment problem that requires special treatment in many models with discrete nonabelian flavor symmetries.<sup>1</sup> In fact,  $\text{CP}^{\mu\tau}$  can be successfully implemented, sometimes accidentally, in models with discrete flavor symmetries [12–17]. More recently, it was shown that maximal  $\theta_{23}$  and maximal Dirac CP phase can be obtained without the explicit imposition of a CP symmetry [18, 19] at the

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<sup>1</sup>See for example refs. [10, 11].

expense of requiring vev alignment and losing the predictions for the Majorana phases. See ref. [20] for a review on  $\text{CP}^{\mu\tau}$  and also on the  $\mu\tau$  interchange symmetry [21–25].

Our main goal here is to show that one can have  $\text{CP}^{\mu\tau}$  symmetry along with a discrete abelian symmetry that ensures a *one-zero texture*.<sup>2</sup> This reduces the number of free parameters in the neutrino mass matrix from five to four to account for the four observables  $\Delta m_{21}^2, \Delta m_{32}^2, \theta_{12}, \theta_{13}$  — the rest are fixed from symmetry — and we obtain a highly predictive scenario where the absolute neutrino mass is fixed and further correlations of parameters appear.

Our approach is a combination of two very different approaches to lepton flavor: (a) texture-zeros that increase predictivity and relate mixing angles with masses [26–30] and (b) symmetries that fully or partly determine the mixing structure independently of the masses [10, 11, 31]. The former usually requires abelian symmetries [32] while the latter requires nonabelian discrete symmetries [10, 11]. The most general cases of texture-zeros has been analyzed recently in ref. [33] where generic texture-zeros are required for the mass matrices of both charged leptons and neutrinos. These cases include the well-studied parallel structures where both mass matrices have the same texture-zeros [34–38]. We refer to ref. [39] for a review. In contrast, residual CP symmetries have also been considered to determine the mixing angles (and CP phases) [40–43, 50–53].

If we give up symmetries that predict mixing angles, it is also possible to use nonabelian flavor symmetries to obtain texture-zeros together with equal elements in the neutrino mass matrix, a scenario known as hybrid texture [44–46], recently generalized in ref. [47]. Non-abelian groups are generally required because one needs noncommuting symmetries. The combination of two noncommuting symmetries cannot be arbitrary and needs to fulfill some compatibility rules so that the whole group closes.<sup>3</sup> In fact, some consistency conditions are required to combine a CP symmetry with a nonabelian discrete flavor group [50–53]. For this reason, we choose the simplest setting where we combine an abelian discrete symmetry with  $\text{CP}^{\mu\tau}$  in a consistent but nontrivial way. As a result, the chosen abelian symmetry will simultaneously be responsible for the diagonal charged lepton masses and for the texture-zero in the neutrino mass matrix.

The outline of this work is as follows: in section 2 we show the two symmetries that will be combined in a consistent way. A useful parametrization of the neutrino mass matrix is shown in section 3 and the possible one-zero textures that are compatible with data are presented in section 4. Section 5 develops an example model and our conclusions can be read in section 6.

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<sup>2</sup>Due to  $\text{CP}^{\mu\tau}$  symmetry some entries are related and only one-zero textures are allowed so that none of these cases correspond to the two-zero textures studied first in ref. [26].

<sup>3</sup>It is not difficult to construct infinite discrete groups when combining noncommuting symmetries with three dimensional representations [48, 49].

## 2 Underlying symmetry

We say the neutrino mass matrix is invariant by  $\text{CP}^{\mu\tau}$  symmetry, or  $\mu - \tau$  reflection [4–6], when

$$M_\nu = \begin{pmatrix} a & d & d^* \\ d & c & b \\ d^* & b & c^* \end{pmatrix}, \quad \text{with real } a, b \text{ and } \text{Im}(d^2 c^*) \neq 0. \quad (2.1)$$

By rephasing we can eliminate either the phase of  $c$  or  $d$  so that we have five real continuous parameters in total. This form for the neutrino mass matrix in the flavor basis (diagonal charged lepton masses) is known to predict maximal  $\theta_{23} = 45^\circ$  and  $\delta = \pm 90^\circ$  at the same time that  $\theta_{13} \neq 0$  is allowed [4–6]. Additionally, the Majorana phases are trivial and four discrete choices for the CP parities are possible [4–6, 9]. These features lead to characteristic predictions for the neutrinoless double beta decay rate and leptogenesis [9].

The mass matrix (2.1) has five real independent parameters to describe five observables:  $\theta_{12}, \theta_{13}, m_1, m_2, m_3$ . One of them — the absolute neutrino mass scale — is unknown. Given the same number of parameters and observables, there is no sharp prediction for the latter if only  $\text{CP}^{\mu\tau}$  is present. We will show in the following that one can have  $\text{CP}^{\mu\tau}$  symmetry along with an abelian symmetry that ensures a *one-zero texture*. With one less parameter we obtain a definite prediction on the absolute neutrino mass. It is clear that  $d$  or  $c$  cannot vanish because the resulting matrix after appropriate rephasing is symmetric by  $\nu_\mu - \nu_\tau$  interchange which leads to the experimentally excluded value of  $\theta_{13} = 0$ .

In order to implement  $\text{CP}^{\mu\tau}$  naturally, it was shown in ref. [9] that the only way we can combine a residual U(1) symmetry in the charged lepton sector and a residual CP symmetry in the neutrino sector with nontrivial CP violation is to consider a U(1) generated by the combination of lepton flavor numbers  $L_\mu - L_\tau$  and  $\text{CP}^{\mu\tau}$  as the CP symmetry. In group theoretical terms, other combinations such as  $L_e - L_\mu$  and  $\text{CP}^{e\mu}$  are allowed but they are not phenomenologically viable.

If we allow the electron flavor to have nontrivial charge and consider  $\mathbb{Z}_n$  instead of U(1), other possibilities arise beginning with  $\mathbb{Z}_8$  [9]. Here we use such a possibility to ensure  $\text{CP}^{\mu\tau}$  symmetry with a texture-zero. We assign  $\mathbb{Z}_8$  charges to the charged leptons  $(e, \mu, \tau)$  as follows

$$T = \begin{pmatrix} -1 & & \\ & \omega_8 & \\ & & \omega_8^3 \end{pmatrix}, \quad \omega_8 = e^{i2\pi/8}. \quad (2.2)$$

This symmetry ensures diagonal charged lepton masses. In contrast, the  $\text{CP}^{\mu\tau}$  symmetry acts as usual on the left-handed neutrino fields  $\nu_{\alpha L}$ ,  $\alpha = e, \mu, \tau$ , as

$$\text{CP}^{\mu\tau} : \quad \nu_{\alpha L} \rightarrow X_{\alpha\beta} \nu_{\beta L}^{cp}, \quad (2.3)$$

where  $cp$  denotes the usual CP conjugation and  $X$  is  $\nu_\mu$ - $\nu_\tau$  interchange,

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (2.4)$$

We can think that these two symmetries —  $\mathbb{Z}_8$  generated by  $T$  and  $\text{CP}^{\mu\tau}$  — initially act on the left-handed lepton doublets  $(L_e, L_\mu, L_\tau)$  before they are spontaneously broken. Then the two symmetries act on the same space and  $\text{CP}^{\mu\tau}$  induces the following automorphism on  $\mathbb{Z}_8$  [50–53]:

$$T \rightarrow XT^*X^{-1} = T^5. \quad (2.5)$$

We also note that the rephasing transformations that preserve  $\mathbb{Z}_8$  in (2.2) and  $\text{CP}^{\mu\tau}$  in (2.3) are of the form

$$L_e \rightarrow \pm L_e, \quad L_\mu \rightarrow e^{i\alpha} L_\mu, \quad L_\tau \rightarrow e^{-i\alpha} L_\tau. \quad (2.6)$$

It is clear that these transformations also preserve the form of the mass matrix in (2.1) and can be used to make  $c$  or  $d$  real. Flavor independent rephasing by  $i$  also preserves the form of the mass matrix (flips the sign of  $a, b$ ) but changes  $\text{CP}^{\mu\tau}$  by a global sign. Hence, only the relative sign of  $a$  and  $b$  is significant.

On the other hand, each quadratic combination  $\bar{\nu}_{\alpha L}^c \nu_{\beta L}$  (Majorana neutrinos) that will give rise to the neutrino mass matrix carries the following  $\mathbb{Z}_8$  charges:

$$\bar{\nu}_{\alpha L}^c \nu_{\beta L} \sim \begin{pmatrix} 1 & \omega_8^5 & \omega_8^{-1} \\ \star & \omega_8^2 & -1 \\ \star & \star & \omega_8^{-2} \end{pmatrix}. \quad (2.7)$$

As all entries carry different charges (including the trivial), we can arrange the appropriate texture-zero in the  $(ee)$  or  $(\mu\tau)$  entry by making the nonzero entries come from the vacuum expectation values of scalars carrying the desired quantum numbers [32]. We give an explicit construction in section 5.

### 3 Parametrization

It was shown in refs. [4–6] that the  $\text{CP}^{\mu\tau}$  symmetric matrix (2.1) can be diagonalized by a matrix of the form

$$U_0 = \begin{pmatrix} u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \\ w_1^* & w_2^* & w_3^* \end{pmatrix}, \quad (3.1)$$

with  $u_i$  real and conventionally positive. The diagonalization performs

$$U_0^\top M_\nu U_0 = \text{diag}(m'_1, m'_2, m'_3), \quad (3.2)$$

where  $m'_i = \pm m_i$ , with  $m_i$  being the neutrino masses. Therefore, the full diagonalizing matrix can be written as

$$U_\nu = U_0 K, \quad (3.3)$$

where  $K$  is a diagonal matrix of 1 or  $i$  depending on the sign on (3.2). We can classify the cases according to the sign of  $m'_i$  or the diagonal entries of  $K^2$  [9] as

$$(+ + +), (- + +), (+ - +), (+ + -). \quad (3.4)$$

There is also the freedom to replace  $U_0$  by  $U_0^*$  in (3.2), together with  $M_\nu \rightarrow M_\nu^*$ . This replacement flips the sign of the Dirac CP phase and the Jarlskog invariant, leaving the rest of observables invariant.

Comparing (3.1) to the standard parametrization of the PMNS matrix and choosing the convention that  $-iw_3 > 0$  we arrive at the parametrization

$$U_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{\pm i}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{\mp i}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3.5)$$

The  $\pm$  sign coincides with the Dirac CP phase given by  $e^{i\delta} = \pm i$  and  $\theta_{23} = 45^\circ$  is fixed by symmetry. Note that the standard parametrization corresponds to  $\text{diag}(1, 1, -1)U_0 \text{diag}(1, 1, \mp i)$ .

If we invert the relation (3.2) by using (3.5) with the top signs, we can write the parameters  $a, b, c, d$  in terms of the neutrino masses and mixing angles:

$$\begin{aligned} a &= c_{13}^2(m'_1 c_{12}^2 + m'_2 s_{12}^2) + m'_3 s_{13}^2, \\ b &= \frac{1}{2}[m'_1 s_{12}^2 + m'_2 c_{12}^2 + s_{13}^2(m'_1 c_{12}^2 + m'_2 s_{12}^2) + m'_3 c_{13}^2], \\ d &= \frac{c_{12}s_{12}c_{13}}{\sqrt{2}}(m'_2 - m'_1) + i \frac{s_{13}c_{13}}{\sqrt{2}}[-m'_3 + m'_1 c_{12}^2 + m'_2 s_{12}^2], \\ c &= \frac{1}{2}[m'_1(s_{12}^2 - c_{12}^2 s_{13}^2) + m'_2(c_{12}^2 - s_{12}^2 s_{13}^2) - m'_3 c_{13}^2] + i c_{12}s_{12}s_{13}(m'_2 - m'_1). \end{aligned} \quad (3.6)$$

Choosing the bottom signs in (3.5) corresponds to taking  $d \rightarrow d^*$  and  $c \rightarrow c^*$ . The phases of  $c, d$  are also convention dependent as they can be transferred from one to the other by the rephasing transformation (2.6). A rephasing invariant CP-odd quantity is

$$\text{Im}(c^* d^2) = \pm \frac{1}{2}(m'_1 - m'_2)(m'_2 - m'_3)(m'_3 - m'_1)s_{13}c_{13}^2 s_{12}c_{12}. \quad (3.7)$$

for both signs in (3.5). This invariant is clearly nonzero for physical values and corresponds to one of the invariants in refs. [54, 55] adapted to the  $\text{CP}^{\mu\tau}$  symmetry case.

We can also note that if we perform a change of basis of  $M_\nu$  only by the first matrix of (3.5), we obtain a real symmetric matrix which can be diagonalized by a real orthogonal matrix. If we compare the trace of  $M_\nu$  and  $M_\nu^2$  in this new basis as well as the determinant, we obtain the following relations:

$$\begin{aligned} m'_1 + m'_2 + m'_3 &= a + 2b, \\ m_1^2 + m_2^2 + m_3^2 &= a^2 + 2(b^2 + |c|^2 + 2|d|^2), \\ m'_1 m'_2 m'_3 &= a(b^2 - |c|^2) - 2b|d|^2 + 2\text{Re}(c^* d^2). \end{aligned} \quad (3.8)$$

This means that we can trade three among  $a, b, |c|, |d|, \text{Im}(c^* d^2)$  by the three neutrino masses  $m_i$  for each choice of CP parities.

Case	$(M_\nu)_{\alpha\beta}=0$	ordering	CP parities	$m_0$	$m_{\beta\beta}$	$\sum m_\nu$
I	$(ee)$	NO	$(-++)$	$4.4 - 9.0$	0	$63 - 74$
II	$(ee)$	NO	$(+ - +)$	$1.1 - 3.9$	0	$59 - 65$
III	$(\mu\tau)$	NO	$(++-)$	$151 - 185$	$142 - 178$	$460 - 561$
IV	$(\mu\tau)$	IO	$(+ - +)$	$15 - 30$	$14.3 - 29.3$	$116 - 148$

**Table 1.** Possibilities for one-zero textures with predictions for the lightest neutrino mass ( $m_0$ ), neutrinoless double beta decay effective mass ( $m_{\beta\beta}$ ) and sum of neutrino masses; all masses are in meV.

## 4 Possible one-zero textures

A texture-zero in the  $(ee)$  or  $(\mu\tau)$  entries of the  $\mathbf{CP}^{\mu\tau}$  symmetric neutrino mass matrix (2.1) is possible depending on the neutrino CP parities and the mass ordering. By using the relations in (3.6), the texture-zero relation essentially fixes the lightest neutrino mass except for the uncertainty in the values of the mixing angles and mass differences.

The solutions are summarized in table 1, where we show the possible values for the mass of the lightest neutrino, the effective neutrinoless double beta decay parameter ( $m_{\beta\beta}$ ) and the sum of neutrino masses. The mixing angles  $\theta_{12}, \theta_{13}$  and the mass differences  $\Delta m_{12}^2, \Delta m_{23}^2$  are taken within  $3\text{-}\sigma$  of the global fit of ref. [7] while the values  $\theta_{23} = 45^\circ$  and  $\delta = \pm\pi/2$  are fixed by symmetry.

We can see that case III is excluded due to the Planck power spectrum limit (95% C.L.) [56],

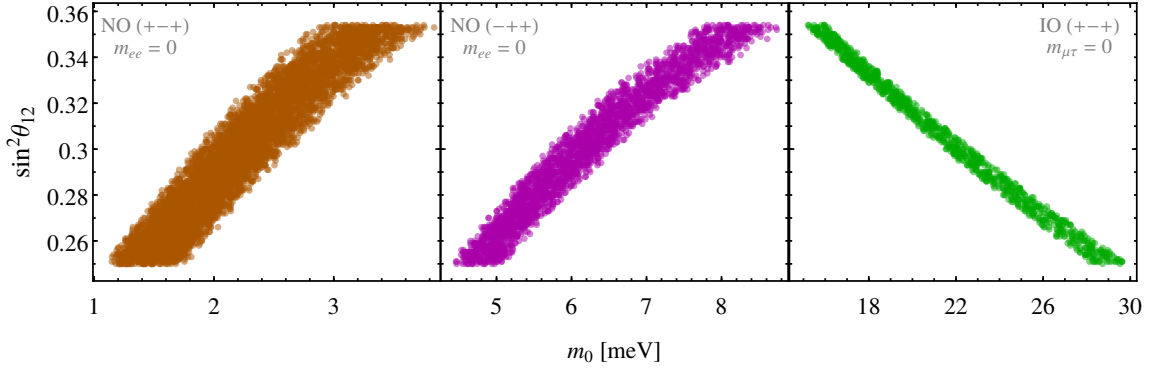
$$\sum_i m_i < 230 \text{ meV}. \quad (4.1)$$

We are left with two cases for the normal ordering (NO) and one case for the inverted ordering (IO). All these cases are also compatible with the latest KamLAND-Zen upper limit for the neutrinoless double beta decay parameter at 90% C.L. [57],

$$m_{\beta\beta} < (61 - 165) \text{ meV}. \quad (4.2)$$

The variation in the latter, comes from the uncertainty in the various evaluations of the nuclear matrix elements. In the future, KamLAND-Zen and EXO-200 experiments will probe the IO region that includes our case IV.

The texture-zero relation  $a = 0$  or  $b = 0$  in (3.6) also leads to a correlation between mixing angles and the lightest neutrino mass when the parameters are allowed to vary within the experimental uncertainties. For the phenomenologically allowed cases, we show this correlation in figure 1 for  $\theta_{12}$ . We can see that the correlation is strong for  $\theta_{12}$  while for  $\theta_{13}$  we have checked that it is only mild. It is clear that a more precise determination of  $\theta_{12}$  will lead to a more precise prediction for the lightest neutrino mass. Concerning the neutrinoless double beta decay rates, this information leads to a testable prediction for  $m_{\beta\beta}$  in case IV but only to a falsifiable prediction for other cases ( $m_{\beta\beta} = 0$ ).



**Figure 1.** Correlation between  $\sin^2 \theta_{12}$  and the lightest neutrino mass  $m_0$  for  $\text{CP}^{\mu\tau}$  symmetric neutrino mass matrix with one-zero textures. The oscillation observables are varied within  $3\text{-}\sigma$  of ref. [7] and  $m_{\alpha\beta}$  denote  $(M_\nu)_{\alpha\beta}$ .

## 5 Model

In order to obtain the  $\text{CP}^{\mu\tau}$  symmetric mass matrix (2.1) with vanishing  $(ee)$  or  $(\mu\tau)$  entries, it is sufficient to introduce scalars carrying  $\mathbb{Z}_8$  charges corresponding to the nonzero entries in (2.7). The absence of appropriate fields will lead to texture-zeros [32].

Let us introduce SM singlet scalars  $\eta_k \sim \omega_8^k$  labelled by their  $\mathbb{Z}_8$  charges,  $k \leq 7$ . We need to know how  $\eta_k$  transforms under  $\text{CP}^{\mu\tau}$ . For that, we just need to infer how  $\eta_1$  transforms. From (2.3), we can see that the lepton doublet  $L_\mu \sim \omega_8$  has the same charge and transforms under  $\text{CP}^{\mu\tau}$  as

$$\text{CP}^{\mu\tau} : L_\mu \rightarrow L_\tau^{cp}, \quad (5.1)$$

where  $L_\tau^{cp} \sim \omega_8^{-3}$ . So we expect

$$\text{CP}^{\mu\tau} : \eta_1 \rightarrow \eta_3^*. \quad (5.2)$$

If we double the charge, we obtain

$$\text{CP}^{\mu\tau} : \eta_2 \rightarrow \eta_6^* \sim \eta_2, \quad (5.3)$$

and the last identification can be made if  $\eta_2$  carries no other quantum number besides  $\mathbb{Z}_8$  [9]. Therefore, the fields  $\eta_1, \eta_3, \eta_2^*, \eta_2$  couple to the appropriate quadratic combination in (2.7) giving rise to the  $(e\tau), (e\mu), (\mu\mu)$  and  $(\tau\tau)$  entries of  $M_\nu$ , respectively.

To avoid the  $(ee)$  combination in (2.7) to acquire a bare coupling,<sup>4</sup> we introduce a  $\mathbb{Z}_4^{B-L}$  symmetry under which the leptons have charge  $-i$  while the scalar  $\eta_k \sim -1$ .<sup>5</sup> If we also introduce the *real* fields  $\eta_0$  and  $\eta_4$ , the dimension five Weinberg operator will come from the nonrenormalizable operators

$$\begin{aligned} & \frac{1}{2} \frac{c_{ee}}{\Lambda^2} \eta_0 L_e H L_e H + \frac{c_{\mu\tau}}{\Lambda^2} \eta_4 L_\mu H L_\tau H + \frac{c_{e\mu}}{\Lambda^2} \eta_3 L_e H L_\mu H + \frac{c_{e\tau}}{\Lambda^2} \eta_1 L_e H L_\tau H \\ & + \frac{1}{2} \frac{c_{\mu\mu}}{\Lambda^2} \eta_2^* L_\mu H L_\mu H + \frac{1}{2} \frac{c_{\tau\tau}}{\Lambda^2} \eta_2 L_\tau H L_\tau H + h.c. \end{aligned} \quad (5.4)$$

<sup>4</sup>In the case of  $(M_\nu)_{\mu\tau} = 0$  we can obtain  $(M_\nu)_{ee}$  from the bare coupling but it will come from a operator which is lower order than the rest.

<sup>5</sup>Since this charge is real, the identification of  $\eta_6^* = \eta_2$  in (5.3) is consistent [9].



Since  $\text{CP}^{\mu\tau}$  symmetry ensures  $c_{e\tau} = c_{e\mu}^*$ ,  $c_{\tau\tau} = c_{\mu\mu}^*$  and real  $c_{ee}, c_{\mu\tau}$ , we obtain the  $\text{CP}^{\mu\tau}$  symmetric mass matrix (2.1) if  $\text{CP}^{\mu\tau}$  is not broken by  $\eta_k$ , i.e.,

$$\langle \eta_3 \rangle = \langle \eta_1 \rangle^* . \quad (5.5)$$

We show in the following that these symmetric vevs are possible.

Finally, the texture-zero in the  $(ee)$  or  $(\mu\tau)$  entry follows if  $\eta_0$  or  $\eta_4$  is absent. In this effective case, the texture-zero is not exact because even if  $\eta_4$  is absent it can be replaced by e.g.  $\eta_0\eta_2^2$  or  $\eta_1^*\eta_3\eta_2$  but it only appears with three  $\eta_k$  fields due to  $\mathbb{Z}_4^{B-L}$  and give entries in the neutrino mass matrix suppressed by  $\langle \eta_k \rangle^2/\Lambda^2$ . Possibly, this suppression can be improved in a specific UV complete model. Some examples of UV completions for the case where the abelian symmetry is  $L_\mu - L_\tau$  can be seen in ref. [9].

The remaining task is to check that the scalar potential involving  $\eta_k$  can be minimized by values conserving  $\text{CP}^{\mu\tau}$ , i.e., obeying (5.5). The potential contains no trilinear terms and the terms that depend on their phases are only quartic:

$$\begin{aligned} V \supset & \lambda_1(\eta_1\eta_3)^2 + \lambda'_1\eta_1^*\eta_3(\eta_1^{*2} + \eta_3^2) + \lambda_2\eta_2^4 + \lambda_3\eta_2^2(\eta_1\eta_3 + \eta_1^*\eta_3^*) \\ & + \lambda_4\eta_0\eta_2^*\eta_1^*\eta_3 + \lambda'_4\eta_0\eta_2(\eta_1^{*2} + \eta_3^2) + h.c., \end{aligned} \quad (5.6)$$

where  $\lambda_1$  is real while the rest are complex. This corresponds to the case where  $\eta_4$  is absent.

If we parametrize  $\langle \eta_k \rangle = u_k e^{i\alpha_k}$ , we can see that the only terms that depend on the combination  $\alpha_{13} \equiv \alpha_1 + \alpha_3$  are the terms with coefficients  $\lambda_1, \lambda'_1, \lambda_3, \lambda'_4$ . The  $\text{CP}^{\mu\tau}$  symmetry corresponds to

$$u_1 \leftrightarrow u_3, \quad \alpha_1 \leftrightarrow -\alpha_3, \quad (5.7)$$

which flips the sign of  $\alpha_{13}$  while the rest are invariant. For the terms with  $\lambda_1$  and  $\lambda_3$ , the dependence is through  $\cos 2\alpha_{13}$  and  $\cos \alpha_{13}$  respectively. The dependence for the terms with  $\lambda'_1$  and  $\lambda'_4$  is only through  $(\eta_1^{*2} + \eta_3^2)$  that have the form

$$u_1^2 \cos(\alpha_{13} + \varphi) + u_3^2 \cos(\alpha_{13} - \varphi). \quad (5.8)$$

This expression depends on  $\alpha_{13}$  only through  $\cos \alpha_{13}$  if  $u_1 = u_3$ . In this case, for all these terms, parameters can be chosen so that  $\alpha_{13} = 0$  is a minimum.

To check that  $u_1 = u_3$  can be achieved, we can gather the terms that do not depend on  $\alpha_{13}$  and write the quadratic contributions for  $u_1$  and  $u_3$ , after taking the minimizing values for other parameters, as

$$A(u_1^2 + u_3^2) + 2Bu_1u_3, \quad (5.9)$$

where  $B$  comes from the  $\lambda_3, \lambda_4$  terms. If we arrange  $A + B < 0$  and  $A - B > 0$ ,  $u_1 - u_3 = 0$  minimizes the quadratic terms in (5.9) and hence the whole potential in the direction orthogonal to  $(u_1, u_3) \sim (1, 1)$ . This checks that the symmetric minimum (5.5) is possible. We have also checked that numerically it is easy to obtain the symmetric minimum.

If  $\eta_4$  is present instead of  $\eta_1$ , the terms with coefficients  $\lambda_4$  and  $\lambda'_4$  are replaced by

$$\lambda_4\eta_2\eta_4\eta_1^*\eta_3 + \lambda'_4\eta_2^*\eta_4(\eta_1^{*2} + \eta_3^2) + h.c. \quad (5.10)$$

We arrive at the same result as before: there are parameter regions where  $\text{CP}^{\mu\tau}$  remains conserved by the vevs.

## 6 Conclusions

We have shown by explicit construction a highly predictive scenario where the neutrino mass matrix is symmetric by  $\text{CP}^{\mu\tau}$  or  $\mu\tau$ -reflection and *additionally* contains one texture-zero in the  $(ee)$  or  $(\mu\tau)$  entry. Besides the usual predictions of  $\text{CP}^{\mu\tau}$  — maximal  $\theta_{23}$ , maximal Dirac CP phase and trivial Majorana phases — we find that only two values for  $m_1$  are possible for normal ordering and only one value for  $m_3$  is possible for the inverted ordering. The NO solutions correspond to  $m_1$  of a few meV and the IO solution has  $m_3$  of around 20 meV. The specific intervals when we allow for the uncertainties in the oscillation parameters can be seen in table 1 together with the possible CP parities, the value for the neutrinoless double beta decay parameter and the sum of neutrino masses. The strong correlation that appears between the solar angle  $\theta_{12}$  and the lightest neutrino mass is shown in figure 1. The IO solution is expected to be tested in the near future by the neutrinoless double beta decay experiments such as KamLAND-Zen and EXO-200 as they reach the IO region. For the solutions with NO, we predict a negligible neutrinoless double beta decay rate as  $m_{\beta\beta} \approx 0$  which can be falsified but will be impossible to confirm. Finally, the possibility of a neutrino mass matrix with  $\text{CP}^{\mu\tau}$  symmetry *simultaneously* with a texture-zero that is enforced by symmetry was first shown here and it is only allowed by combining in a non-usual way a discrete abelian symmetry at least as large as  $\mathbb{Z}_8$  and  $\text{CP}^{\mu\tau}$ .

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